

Study of $B_s \rightarrow \eta_c(J/\psi)D$ decay with perturbative QCD approach

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Abstract

The W -exchange process $B_s \rightarrow \eta_c(J/\psi)D$ is studied with the perturbative QCD approach. Three kinds of wave functions for B_s meson and two forms of wave functions for charmonium are considered. It is estimated that branching ratios for $B_s \rightarrow \eta_c \bar{D}$, $\eta_c D$, $J/\psi \bar{D}$, $J/\psi D$ decays are the order of 10^{-7} , 10^{-8} , 10^{-8} , 10^{-9} , respectively, where the largest uncertainty is from wave functions. There is a possibility for measuring these decay in the near future.

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The first evidence for B_s production in e^+e^- annihilation at the $\Upsilon(5S)$ resonance was found by the CLEO collaboration [1]. Belle has accumulated 121 fb^{-1} data at the $\Upsilon(5S)$ resonance, including $7.1 \times 10^6 B_s \bar{B}_s$ pairs [2]. It is estimated that some $5.9 \times 10^8 B_s$ mesons in the dataset of 5 ab^{-1} at the $\Upsilon(5S)$ resonance in a New Snowmass Year (about 10^7 seconds of actual annual running time [3]) will be collected at the high luminosity e^+e^- asymmetric SuperKEKB [4]. More and more B_s decays will have subjected the Standard Model and new physics to a series of increasingly stringent tests, through observables such as branching ratios, CP-violationg asymmetries and kinematic distributions.

In the naive spectator model, the general properties of the B_s meson parallel those of the $B_{u,d}$ mesons. The close correspondences between B_s and $B_{u,d}$ mesons allow for sensitive tests of hadronic models [2]. Hadronic B decays are complicated on account of strong interaction effects, meanwhile, they will have provided a great opportunity to study perturbative and nonperturbative QCD. For nonleptonic two-body B decays, the low-energy effective Hamiltonian ansatz and the factorization hypothesis are commonly used. In recent years, several attractive methods have been proposed and widely used to evaluate the hadronic matrix elements (where the local operators in the effective Hamiltonian are sandwiched between initial and final hadron states considered) based on an expansion in the QCD coupling constant α_s/π and in the power Λ_{QCD}/m_Q (where Λ_{QCD} and m_Q are the QCD characteristic scale and the mass of heavy quark Q , respectively), such as the QCD factorization [5], perturbative QCD method (pQCD) [6], soft and collinear effective theory (SCET) [7], etc.

Using the operator product expansion and renormalization group equation, the low energy effective Hamiltonian for the $B_s \rightarrow \eta_c(J/\psi)D$ decay can be written as [8]:

$$\begin{aligned} \mathcal{H}_{eff} = & \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} \left\{ C_1 (\bar{b}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta s_\beta)_{V-A} + C_2 (\bar{b}_\alpha c_\beta)_{V-A} (\bar{u}_\beta s_\alpha)_{V-A} \right\} \\ & + \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} \left\{ C_1 (\bar{b}_\alpha u_\alpha)_{V-A} (\bar{c}_\beta s_\beta)_{V-A} + C_2 (\bar{b}_\alpha u_\beta)_{V-A} (\bar{c}_\beta s_\alpha)_{V-A} \right\} + \text{H.c.}, \end{aligned} \quad (1)$$

where G_F is the Fermi coupling constant. The Cabibbo-Kobayashi-Maskawa (CKM) matrix factors $V_{cb}^* V_{us}$ (or $V_{ub}^* V_{cs}$) and the Wilson coefficients $C_{1,2}$ describe the strengths of the local four-quark operators in the effective Hamiltonian. α and β are $SU(3)$ color indices. $(\bar{q}q')_{V-A} = \bar{q}\gamma_\mu(1 - \gamma_5)q'$. The Wilson coefficients, which incorporate the physics contributions from high scales, have been calculated to the next-to-leading order in the perturbation theory and evolved to a characteristic scale with the renormalization group equation [8]. The essential problem obstructing the calculation of nonleptonic decay amplitudes is how to evaluate the

hadronic matrix elements of the local operators properly and accurately.

Based on the principle of color transparency [10] and factorization scheme, the phenomenological treatment of the hadronic matrix elements for the W -exchange processes $B_s \rightarrow \eta_c(J/\psi)D$ is the same as that for pure annihilation topologies. Although the annihilation amplitude is formally power suppressed by Λ_{QCD}/m_b with the QCDF power counting conventions [5], its contribution is indispensable for realistic B -meson decays [6]. The comprehensive analysis of $B_{u,d} \rightarrow PP, PV$ decays without taking into account the annihilation contributions is generally of poor quality [11] (P and V denote the pseudoscalar and vector mesons, respectively). Study of $B_s \rightarrow \eta_c(J/\psi)D$ decay will help to improve our understanding of the annihilation effects.

Analogous to the analysis for hard exclusive scattering amplitudes [12], the hadronic matrix element is commonly expressed as a convolution of scattering kernels with the universal wave functions of the participating hadrons [5, 6], where nonperturbative dynamics either cancel or is absorbed into hadron wave functions (WFs). However, the annihilation effects in the collinear approximation exhibit endpoint singularities (ES) for charmless mesonic B decays, displaying inconsistency of the QCDF formula [5].

To deal with ES in convolution integrals, many attempts will have been made.

(1) A phenomenological parameterization of ES in annihilation contributions is originally proposed by QCDF itself [5], which not only introduces uncertainties in the QCDF's prediction of observables, especially for annihilation dominated processes [11], but also provide no constraint on magnitudes of strong phases relevant to CP violation.

(2) ES is removed by separating the physics at different momentum scales using the zero-bin subtraction to avoid double counting of soft degrees of freedom in SCET [13], while the imaginary part of the amplitude is also dropped at the leading power in $\alpha_s \Lambda_{\text{QCD}}/m_b$.

(3) The infrared finite gluon propagator and running coupling constant [14], or/and Cutkosky rules [15] for the quark propagators, are used to serve as a natural cutoff, which has already been applied to B decays into two mesons [16–18]. However, it is claimed [18] that different predictions on branching ratios can be obtained with different solutions of the Schwinger-Dyson equations for gluon propagator and coupling constant due to different truncations and approximations.

(4) By keeping the parton transverse momentum k_T , and employing the Sudakov factors to smear the double logarithm in QCD radiative corrections and to suppress the endpoint

contribution of hadron wave functions in small transverse momentum region, ES in collinear approximation is eliminated with the pQCD approach, and the strong phases are perturbatively calculated [6]. An example is the recently renewed study on the pure annihilation decays $B_s \rightarrow \pi^+\pi^-$ and $B_d \rightarrow K^+K^-$ with the pQCD approach [19] which are in good agreement with the CDF and LHCb measurements.

Despite disputes as to which one of above treatments is more effective than others, we will study the $B_s \rightarrow \eta_c(J/\psi)D$ decays with the pQCD approach to give a rough estimate of their branching ratios. Based on k_T factorization, the typical expression for the decay amplitudes with the pQCD approach can be expressed as

$$\int \mathbf{d}k C(t)H(k,t)\Phi(k)e^{-S} \quad (2)$$

where C , H , Φ , and e^{-S} are Wilson coefficient, hard-scattering kernel, hadron WFs, and Sudakov factor, respectively. The typical scale t depends on topology and process. For convenience, the kinematics variables are described by light cone coordinate. The momenta of the valence quarks and hadrons in the rest frame of the B_s meson are defined as follows:

$$p_1 = \frac{m_1}{\sqrt{2}}(1, 1, \vec{0}_\perp), \quad (3)$$

$$p_2 = (\eta_2^+, \eta_2^-, \vec{0}_\perp), \quad (4)$$

$$p_3 = (\eta_3^-, \eta_3^+, \vec{0}_\perp), \quad (5)$$

$$k_i = x_i p_i + (0, 0, \vec{k}_{i\perp}), \quad (6)$$

$$\eta_i^\pm = \frac{E_i \pm p}{\sqrt{2}}, \quad (7)$$

$$\epsilon_2 = \frac{1}{m_2}(\eta_2^+, -\eta_2^-, \vec{0}_\perp), \quad (8)$$

where the subscript $i = 1, 2, 3$ refers to B_s , $\eta_c(J/\psi)$, D meson. k_i , $\vec{k}_{i\perp}$, x_i are the momentum, transverse momentum and longitudinal momentum fraction of light quark of meson, respectively. ϵ_2 is the longitudinal polarization vector of J/ψ meson. In the rest frame of B_s meson, E_i is the energy of particle i , and p is the common momentum of final state.

The basic input element in Eq.(2) — WFs — is defined by the nonlocal bilinear quark operator matrix element [21].

$$\langle 0 | \bar{b}_\alpha(0) s_\beta(z) | B_s(p_1) \rangle = \frac{-i}{\sqrt{2N_c}} \int \mathbf{d}^4 k_1 e^{-ik_1 \cdot z} \left\{ \left[\frac{\not{n}_-}{\sqrt{2}} \phi_B^+(k_1) + \frac{\not{n}_+}{\sqrt{2}} \phi_B^-(k_1) \right] (\not{p}_1 + m_B) \gamma_5 \right\}_{\beta\alpha}, \quad (9)$$

$$\langle J/\psi(p_2) | \bar{c}_\alpha(0) c_\beta(z) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int \mathbf{d}^4 k_2 e^{+ik_2 \cdot z} \not{\epsilon}_2 \left[m_{J/\psi} \phi_\psi^L(k_2) + \not{p}_2 \phi_\psi^t(k_2) \right]_{\beta\alpha}, \quad (10)$$

$$\langle \eta_c(p_2) | \bar{c}_\alpha(0) c_\beta(z) | 0 \rangle = \frac{-i}{\sqrt{2N_c}} \int \mathbf{d}^4 k_2 e^{+ik_2 \cdot z} \left\{ \gamma_5 \left[\not{p}_2 \phi_{\eta_c}^v(k_2) + m_{\eta_c} \phi_{\eta_c}^s(k_2) \right] \right\}_{\beta\alpha}, \quad (11)$$

$$\langle D(p_3) | \bar{c}_\alpha(0) u_\beta(z) | 0 \rangle = \frac{-i}{\sqrt{2N_c}} \int \mathbf{d}^4 k_3 e^{+ik_3 \cdot z} \left[\gamma_5 (\not{p}_3 + m_D) \phi_D(k_3) \right]_{\beta\alpha}, \quad (12)$$

where $N_c = 3$ is the color number. n_- and n_+ are null vectors, and $n_+ \cdot n_- = 1$.

Here, the distribution amplitude of D meson given in [20] is used,

$$\phi_D(x) = \frac{f_D}{2\sqrt{2N_c}} 6x\bar{x} [1 + C_D(1 - 2x)], \quad (13)$$

where $\bar{x} = 1 - x$. f_D is the decay constant. C_D is a shape parameter.

For WFs of η_c and J/ψ mesons, $\phi_{\eta_c}^v$ and ϕ_ψ^L are twist-2; $\phi_{\eta_c}^s$ and ϕ_ψ^t are twist-3. They can be extracted from the Schrödinger state with dynamical potentials [21, 22]. We will consider two kinds of WFs corresponding to harmonic-oscillator and Coulomb potentials. Their expressions are listed in [22]. One is the harmonic-oscillator (O) type

$$\phi_\psi^L(x, b) = \frac{f_{J/\psi}}{2\sqrt{2N_c}} N_\psi^L x\bar{x} \exp\left\{ -\frac{m_c}{\omega} x\bar{x} \left[\left(\frac{x - \bar{x}}{2x\bar{x}} \right)^2 + \omega^2 b^2 \right] \right\}, \quad (14)$$

$$\phi_\psi^t(x, b) = \frac{f_{J/\psi}}{2\sqrt{2N_c}} N_\psi^t (x - \bar{x})^2 \exp\left\{ -\frac{m_c}{\omega} x\bar{x} \left[\left(\frac{x - \bar{x}}{2x\bar{x}} \right)^2 + \omega^2 b^2 \right] \right\}, \quad (15)$$

$$\phi_{\eta_c}^v(x, b) = \frac{f_{\eta_c}}{2\sqrt{2N_c}} N_{\eta_c}^v x\bar{x} \exp\left\{ -\frac{m_c}{\omega} x\bar{x} \left[\left(\frac{x - \bar{x}}{2x\bar{x}} \right)^2 + \omega^2 b^2 \right] \right\}, \quad (16)$$

$$\phi_{\eta_c}^s(x, b) = \frac{f_{\eta_c}}{2\sqrt{2N_c}} N_{\eta_c}^s \exp\left\{ -\frac{m_c}{\omega} x\bar{x} \left[\left(\frac{x - \bar{x}}{2x\bar{x}} \right)^2 + \omega^2 b^2 \right] \right\}. \quad (17)$$

The other is the Coulomb (C) type

$$\phi_\psi^L(x, b) = \frac{f_{J/\psi}}{2\sqrt{2N_c}} N_\psi^L \frac{(x\bar{x})^2 m_c b}{\sqrt{1 - 4x\bar{x}(1 - v^2)}} K_1(m_c b \sqrt{1 - 4x\bar{x}(1 - v^2)}), \quad (18)$$

$$\phi_\psi^t(x, b) = \frac{f_{J/\psi}}{2\sqrt{2N_c}} N_\psi^t \frac{(x - \bar{x})^2 x\bar{x} m_c b}{\sqrt{1 - 4x\bar{x}(1 - v^2)}} K_1(m_c b \sqrt{1 - 4x\bar{x}(1 - v^2)}), \quad (19)$$

$$\phi_{\eta_c}^v(x, b) = \frac{f_{\eta_c}}{2\sqrt{2N_c}} N_{\eta_c}^v \frac{(x\bar{x})^2 m_c b}{\sqrt{1 - 4x\bar{x}(1 - v^2)}} K_1(m_c b \sqrt{1 - 4x\bar{x}(1 - v^2)}), \quad (20)$$

$$\phi_{\eta_c}^s(x, b) = \frac{f_{\eta_c}}{2\sqrt{2N_c}} N_{\eta_c}^s \frac{x\bar{x} m_c b}{\sqrt{1 - 4x\bar{x}(1 - v^2)}} K_1(m_c b \sqrt{1 - 4x\bar{x}(1 - v^2)}). \quad (21)$$

where $f_{J/\psi}$ and f_{η_c} are decay constants. m_c is the mass of c quark. b is the conjugate variable of the transverse momentum. $\omega \approx 0.6$ GeV and $v \approx 0.3$ [22] are shape parameters. $N_\psi^{L,t}$ and $N_{\eta_c}^{v,s}$ are the normalization constants. The normalization conditions are

$$\int_0^1 \mathbf{d}x \phi_\psi^{L,t}(x, 0) = \frac{f_{J/\psi}}{2\sqrt{2N_c}}, \quad (22)$$

$$\int_0^1 \mathbf{d}x \phi_{\eta_c}^{v,s}(x, 0) = \frac{f_{\eta_c}}{2\sqrt{2N_c}}. \quad (23)$$

For WFs of B_s meson, there are two scalar compositions ϕ_B^+ and ϕ_B^- . Neglecting three-particle amplitudes, the equation of motion for ϕ_B^\pm is [23, 24]

$$\phi_B^+(x) + x \phi_B^{-'}(x) = 0. \quad (24)$$

The relation of Eq.(24) is sometimes referred to as “Wandzura-Wilczek relation” [25]. It is helpful in constraining models for WFs, which leads to $\phi_B^+(x)$ vanished at the endpoint and $\phi_B^-(x) = \mathcal{O}(1)$ for $x \rightarrow 0$ [26]. Here we will investigate three models of WFs for B_s meson. The first one is the exponential (GN) type suggested in [27], i.e.,

$$\phi_{B_s}^{\text{GN}+}(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{GN}}^+ x \exp\left[-\frac{x m_{B_s}}{\omega_{\text{GN}}}\right] \frac{1}{1 + (b \omega_{\text{GN}})^2}, \quad (25)$$

$$\phi_{B_s}^{\text{GN}-}(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{GN}}^- \exp\left[-\frac{x m_{B_s}}{\omega_{\text{GN}}}\right] \frac{1}{1 + (b \omega_{\text{GN}})^2}. \quad (26)$$

The second one is the Gaussian (KLS) type proposed in [28, 29], i.e.,

$$\phi_{B_s}^{\text{KLS}+}(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{KLS}}^+ x^2 \bar{x}^2 \exp\left[-\frac{1}{2}\left(\frac{x m_{B_s}}{\omega_{\text{KLS}}}\right)^2 - \frac{1}{2}\omega_{\text{KLS}}^2 b^2\right], \quad (27)$$

$$\begin{aligned} \phi_{B_s}^{\text{KLS}-}(x, b) = & \frac{f_{B_s}}{2\sqrt{2N_c}} N_{\text{KLS}}^- \exp\left[-\frac{1}{2}\omega_{\text{KLS}}^2 b^2\right] \left\{ \exp\left[-\frac{1}{2}\left(\frac{x m_{B_s}}{\omega_{\text{KLS}}}\right)^2\right] (m_{B_s}^2 \bar{x}^2 + 2\omega_{\text{KLS}}^2) \right. \\ & \left. + \sqrt{2\pi} m_{B_s} \omega_{\text{KLS}} \text{Erf}\left(\frac{x m_{B_s}}{\sqrt{2}\omega_{\text{KLS}}}\right) + C_{\text{KLS}} \right\}, \end{aligned} \quad (28)$$

where the constant C_{KLS} is chosen so that $\phi_{B_s}^{\text{KLS}-}(1, b) = 0$. The third one is the KKQT type derived by QCD equation of motion and heavy-quark symmetry constraint [24, 30], i.e.,

$$\phi_{B_s}^{\text{KKQT}+}(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} \frac{2x}{\omega_{\text{KKQT}}^2} \theta(y) J_0(m_{B_s} b \sqrt{xy}), \quad (29)$$

$$\phi_{B_s}^{\text{KKQT}-}(x, b) = \frac{f_{B_s}}{2\sqrt{2N_c}} \frac{2y}{\omega_{\text{KKQT}}^2} \theta(y) J_0(m_{B_s} b \sqrt{xy}), \quad (30)$$

where $y = \omega_{\text{KKQT}} - x$.

In Eq.(25—30), f_{B_s} is the decay constant. ω_i is the shape parameters. The normalization conditions is

$$\int_0^1 \phi_{B_s}^\pm(x, 0) \mathbf{d}x = \frac{f_{B_s}}{2\sqrt{2N_c}}. \quad (31)$$

Within the pQCD framework, the Feynman diagrams for $B_s \rightarrow \eta_c D$ decay are shown in FIG.1, where (a) and (b) are non-factorizable topologies, (c) and (d) are factorizable topologies. After a straightforward calculation with the master formula of Eq.(2), the decay amplitudes can be written as follows

$$\mathcal{A}(B_s \rightarrow \eta_c D) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} \sum_{i=a,b,c,d} \mathcal{A}_i, \quad (32)$$

The expressions of \mathcal{A}_i are collected in APPENDIX. From the expressions, we can clearly see that both $\phi_{B_s}^+$ and $\phi_{B_s}^-$ contribute to the decay amplitudes. The branching ratios in the B_s meson rest frame can be written as:

$$\mathcal{BR}(B_s \rightarrow \eta_c D) = \frac{\tau_{B_s}}{8\pi} \frac{p}{m_{B_s}^2} |\mathcal{A}(B_s \rightarrow \eta_c D)|^2, \quad (33)$$

where p is the center-of-mass momentum.

The input parameters in our numerical calculation are collected in TABLE. I. If not specified explicitly, we shall take their central values as default input.

Our study show that (1) contributions of FIG.1 (a-c) can provide large strong phases, which is consistent with pQCD's statement [6]. The interference between factorizable diagrams FIG.1 (c) and (d) is destructive, which is, by and large, in agreement with previous pQCD's estimate (for example, see [34]). The strong phase difference between FIG.1 (c) and (d) is independent of model of WFs for B_s meson. The main contribution is from non-factorizable diagram FIG.1 (b). (2) The dominant contribution is from $\alpha_s/\pi \leq 0.2$ region, implying that despite the small phase space, these processes are calculated with perturbative theory due to hard gluon exchange, where the gluon virtuality scales as $k_g^2 > (2m_c)^2$. (3) There is very strong interference between contributions of WFs ϕ_B^+ and ϕ_B^- , between contributions of twist-2 and twist-3 WFs for $\eta_c(J/\psi)$ mesons. Contribution with only twist-3 WFs for $\eta_c(J/\psi)$ mesons (where twist-2 WFs is zero and twist-3 WFs is nonzero) is less than 30%.

Our numerical results are shown in TABLE. II, where the first uncertainty comes from the WF shape parameter of B_s meson, i.e., $\omega_{\text{GN}} = 0.45 \pm 0.10$ GeV in Eq.(25—26), $\omega_{\text{KLS}} = 0.45 \pm 0.10$ GeV in Eq.(27—28) and $\omega_{\text{KKQT}} = 0.25 \pm 0.10$ in Eq.(29—30); the second uncertainty comes from the WF shape parameter of $J/\psi(\eta_c)$ meson, i.e., $\omega = 0.6 \pm 0.1$ GeV in Eq.(14—17) and $v = 0.3 \pm 0.1$ in Eq.(18—21); the third uncertainty comes from the WF shape parameter of D meson, i.e., $C_D = 0.7 \pm 0.1$ in Eq.(13); the last uncertainty comes from the choice of hard scales $(1 \pm 0.1)t_i$ in Eq.(E11—E12). In addition, decay constants f_D , $f_{J/\psi}$, f_{η_c} , f_{B_s} bring some 10% uncertainty to the branching ratio.

From the numbers in TABLE. II, we can clearly see (1) branching ratios are sensitive to the choice of shape parameter and model of hadronic WFs for B_s and $\eta_c(J/\psi)$ mesons, relative to the choice of hard scale. It is also found that all branching ratios decrease with the increasing shape parameter of hadronic WFs for B_s and $\eta_c(J/\psi)$ mesons. (2) Due to CKM

factors $|V_{cb}^*V_{us}| > |V_{ub}^*V_{cs}|$, there is hierarchic structure $\mathcal{BR}(B_s \rightarrow J/\psi \bar{D}) > \mathcal{BR}(B_s \rightarrow J/\psi D)$ and $\mathcal{BR}(B_s \rightarrow \eta_c \bar{D}) > \mathcal{BR}(B_s \rightarrow \eta_c D)$. Besides, uncertainty ($\sim 30\%$) from $V_{ub}^*V_{cs}$ is much larger than that ($\sim 5\%$) from $V_{cb}^*V_{us}$. (3) Due to $m_{J/\psi} > m_{\eta_c}$ and the orbital angular momentum $L_{J/\psi \bar{D}(D)} > L_{\eta_c \bar{D}(D)}$, the phase space for $B_s \rightarrow J/\psi \bar{D}(D)$ decay is tighter than that for $B_s \rightarrow \eta_c \bar{D}(D)$ decay. With the same input, there are relations $\mathcal{BR}(B_s \rightarrow \eta_c \bar{D}) > \mathcal{BR}(B_s \rightarrow J/\psi \bar{D})$ and $\mathcal{BR}(B_s \rightarrow \eta_c D) > \mathcal{BR}(B_s \rightarrow J/\psi D)$. (4) Branching ratios for $B_s \rightarrow \eta_c \bar{D}$, $\eta_c D$, $J/\psi \bar{D}$, $J/\psi D$ decays are the order of 10^{-7} , 10^{-8} , 10^{-8} , 10^{-9} , respectively.

The corresponding U -spin process $B_d \rightarrow \eta_c(J/\psi) \bar{D}(D)$ has been studied [35–38]. In [35], it is argued that if the intrinsic charm inside B meson is not much less than 1%, branching ratio for $\bar{B}^0(b\bar{d}c\bar{c}) \rightarrow \eta_c(J/\psi)D$ decay is $\sim 10^{-4}$, which is larger than the present experimental upper limit $< 1.3 \times 10^{-5}$ [9]. Based on collinear factorization scenario, the $B_d \rightarrow \eta_c(J/\psi)D$ decay is investigated in [36] with the approach for exclusive processes [12], where narrow δ -function like WFs are used. The small overlapping among WFs results in branching ratio being about $10^{-7} \sim 10^{-8}$ [36]. This issue is renewed in [37] with pQCD approach in the framework of k_T factorization. By keeping the parton transverse momentum and taking the WFs for $c\bar{c}$ final states given in [21], it is found that branching ratio for $B_d \rightarrow \eta_c(J/\psi)D$ decay is about $10^{-5} \sim 10^{-7}$ [37]. Considering the final state interactions, branching ratio for $B_d \rightarrow J/\psi \bar{D}$ is estimated to be $10^{-5} \sim 10^{-6}$ [38]. The results in [36, 37] have similar hierarchic structure due to kinematics and dynamics, i.e., $\mathcal{BR}(B_d \rightarrow \eta_c D) > \mathcal{BR}(B_d \rightarrow J/\psi D)$. The method used in our study is the same as [37], and similar WFs for $\eta_c(J/\psi)$ is employed (in our study, the small relativistic corrections to the WFs are neglected and two types of WFs are considered). A consistent estimation is obtained between ours and [37], using the rate $\frac{\mathcal{BR}(B_s \rightarrow c\bar{c}\bar{D})}{\mathcal{BR}(B_d \rightarrow c\bar{c}\bar{D})} \propto \frac{|V_{cb}^*V_{us}|^2}{|V_{cb}^*V_{ud}|^2} \propto \lambda^2 \sim \mathcal{O}(10^{-2})$.

It is well known that the pure annihilation process $B_s \rightarrow \pi^+\pi^-$ with branching ratio $\sim \mathcal{O}(10^{-7})$ [39] and pure leptonic rare decay $B_s \rightarrow \mu^+\mu^-$ with branching ratio $\sim \mathcal{O}(10^{-9})$ [40] have recently been measured at hadron collider, due to the fact that there have accumulated much data and that detectors sitting at LHC and Tevatron colliders have excellent performance on the final charged particles. We believe that $B_s \rightarrow \eta_c(J/\psi)D$ decay could be accessible experimentally in the near future, because (1) their branching ratio is the same order as (sometimes larger than) that for $B_s \rightarrow \pi^+\pi^-$, $\mu^+\mu^-$ decays. (2) The final D meson can be tagged by charged kaon and/or pion, while tracks of both K^\pm and π^\pm are clearly seen by sensitive detectors. Besides, signal of $\eta_c(J/\psi)$ meson is easily identified by its nar-

row peak in the invariant mass distribution. For example, the LHCb has measured many B_s decay into final states containing a charmonium, such as $B_s \rightarrow J/\psi K^+ K^-$ [41], $J/\psi \bar{K}^{*0}$ [42], $J/\psi f_0(980)$ [43], $J/\psi K_s^0$ [44] (3) More and more B_s data will be accumulated with the running of LHC and advancing SuperKEKB. It seems to exist a realistic possibility to study rare decays with branching ratio $\sim \mathcal{O}(10^{-9})$.

In summary, we study the pure weak annihilation process $B_s \rightarrow \eta_c(J/\psi)D$ decay with pQCD approach. ES disappear as expected by keeping the parton transverse momentum. The largest uncertainty in our result is mainly from QCD's dynamical property of hadron. Branching ratio for $B_s \rightarrow \eta_c(J/\psi)D$ decay depends strongly on model of WFs for B_s and $\eta_c(J/\psi)$ meson. There are some other uncertainties considered here, such as the high order corrections, the effects of final states interaction, and so on. Our estimate show that branching ratios for $B_s \rightarrow \eta_c \bar{D}$, $\eta_c D$, $J/\psi \bar{D}$, $J/\psi D$ decays are the order of 10^{-7} , 10^{-8} , 10^{-8} , 10^{-9} , respectively. They could be measured in the near future.

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Appendix A: The amplitudes for $B_s \rightarrow J/\psi \bar{D}$ decay

$$\mathcal{A}(B_s \rightarrow J/\psi \bar{D}) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} \sum_{i=a,b,c,d} \mathcal{A}_i \quad (\text{A1})$$

$$\begin{aligned} i\mathcal{A}_a = & \frac{32\pi^2 C_F}{\sqrt{2N}} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\ & \times \alpha_s(t_a) C_1(t_a) e^{-S_B - S_\psi - S_D} H_{ab}(\alpha, \beta_a, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\ & \times \left\{ \phi_B^+(x_1, b_1) \phi_\psi^L(x_2, b_2) \eta_2^+ [m_b \eta_3^- + \sqrt{2}\eta(x_1 - x_2) + \sqrt{2}m_3^2(x_1 - x_3)] \right. \\ & - \phi_B^-(x_1, b_1) \phi_\psi^L(x_2, b_2) \eta_2^- [m_b \eta_3^+ + \sqrt{2}\eta(x_1 - x_2) + \sqrt{2}m_3^2(x_1 - x_3)] \\ & + \phi_B^+(x_1, b_1) \phi_\psi^t(x_2, b_2) m_2 m_3 \left[m_b + \frac{1}{2}m_1 x_1 - \frac{1}{\sqrt{2}}\eta_2^+ x_2 - \frac{1}{\sqrt{2}}\eta_3^+ x_3 \right] \\ & \left. - \phi_B^-(x_1, b_1) \phi_\psi^t(x_2, b_2) m_2 m_3 \left[m_b + \frac{1}{2}m_1 x_1 - \frac{1}{\sqrt{2}}\eta_2^- x_2 - \frac{1}{\sqrt{2}}\eta_3^- x_3 \right] \right\}, \quad (\text{A2}) \end{aligned}$$

$$\begin{aligned}
i\mathcal{A}_b = & \frac{32\pi^2 C_F}{\sqrt{2N}} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\
& \times \alpha_s(t_b) C_1(t_b) e^{-S_B - S_\psi - S_D} H_{ab}(\alpha, \beta_b, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \sqrt{2} m_1 p \phi_\psi^L(x_2, b_2) \left[\eta_3^+ \phi_B^+(x_1, b_1) + \eta_3^- \phi_B^-(x_1, b_1) \right] (x_1 - \bar{x}_3) \right. \\
& + m_2 m_3 \phi_B^+(x_1, b_1) \phi_\psi^t(x_2, b_2) \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \left. - m_2 m_3 \phi_B^-(x_1, b_1) \phi_\psi^t(x_2, b_2) \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{A3}
\end{aligned}$$

$$\begin{aligned}
i\mathcal{A}_c = & \frac{-8\pi^2 C_F}{N} m_1 p f_{B_s} \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty b_3 \mathbf{d}b_3 \\
& \times \alpha_s(t_c) \left[C_1(t_c) + N C_2(t_c) \right] e^{-S_\psi - S_D} H_{cd}(\alpha, \beta_c, b_2, b_3) \phi_D(x_3, b_3) \\
& \times \left\{ \phi_\psi^L(x_2, b_2) \left[m_1^2 - (m_1^2 - m_3^2) x_2 \right] - 2 m_2 m_3 x_2 \phi_\psi^t(x_2, b_2) \right\}, \tag{A4}
\end{aligned}$$

$$\begin{aligned}
i\mathcal{A}_d = & \frac{8\pi^2 C_F}{N} m_1 p f_{B_s} \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty b_3 \mathbf{d}b_3 \\
& \times \alpha_s(t_d) \left[C_1(t_d) + N C_2(t_d) \right] e^{-S_\psi - S_D} H_{cd}(\alpha, \beta_d, b_3, b_2) \\
& \times \phi_D(x_3, b_3) \phi_\psi^L(x_2, b_2) \left\{ m_1^2 + m_3 m_c - (m_1^2 - m_2^2) x_3 \right\}, \tag{A5}
\end{aligned}$$

Appendix B: The amplitudes for $B_s \rightarrow J/\psi D$ decay

$$\mathcal{A}(B_s \rightarrow J/\psi D) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} \sum_{i=a,b,c,d} \mathcal{A}_i \tag{B1}$$

$$\begin{aligned}
i\mathcal{A}_a = & \frac{32\pi^2 C_F}{\sqrt{2N}} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\
& \times \alpha_s(t_a) C_1(t_a) e^{-S_B - S_\psi - S_D} H_{ab}(\alpha, \beta_a, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \phi_B^+(x_1, b_1) \phi_\psi^L(x_2, b_2) \eta_3^+ \left[\sqrt{2} m_1 p (x_1 - x_3) - m_b \eta_2^- \right] \right. \\
& + \phi_B^-(x_1, b_1) \phi_\psi^L(x_2, b_2) \eta_3^- \left[\sqrt{2} m_1 p (x_1 - x_3) + m_b \eta_2^+ \right] \\
& - \phi_B^+(x_1, b_1) \phi_\psi^t(x_2, b_2) m_2 m_3 \left[\frac{1}{2} m_1 x_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \left. + \phi_B^-(x_1, b_1) \phi_\psi^t(x_2, b_2) m_2 m_3 \left[\frac{1}{2} m_1 x_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{B2}
\end{aligned}$$

$$i\mathcal{A}_b = \frac{32\pi^2 C_F}{\sqrt{2N}} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3$$

$$\begin{aligned}
& \times \alpha_s(t_b) C_1(t_b) e^{-S_B - S_\psi - S_D} H_{ab}(\alpha, \beta_b, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \sqrt{2} \phi_\psi^L(x_2, b_2) \left[\eta_2^+ \phi_B^+(x_1, b_1) - \eta_2^- \phi_B^-(x_1, b_1) \right] \left[\eta(x_1 - \bar{x}_2) + m_3^2(x_1 - \bar{x}_3) \right] \right. \\
& \quad - m_2 m_3 \phi_B^+(x_1, b_1) \phi_\psi^t(x_2, b_2) \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \quad \left. + m_2 m_3 \phi_B^-(x_1, b_1) \phi_\psi^t(x_2, b_2) \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{B3}
\end{aligned}$$

$$\mathcal{A}_c = -\mathcal{A}_d(B_s \rightarrow J/\psi \bar{D}), \tag{B4}$$

$$\mathcal{A}_d = -\mathcal{A}_c(B_s \rightarrow J/\psi \bar{D}) \tag{B5}$$

Appendix C: The amplitudes for $B_s \rightarrow \eta_c \bar{D}$ decay

$$\mathcal{A}(B_s \rightarrow \eta_c \bar{D}) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} \sum_{i=a,b,c,d} \mathcal{A}_i \tag{C1}$$

$$\begin{aligned}
\mathcal{A}_a = & \frac{32\pi^2 C_F}{\sqrt{2N}} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\
& \times \alpha_s(t_a) C_1(t_a) e^{-S_B - S_{\eta_c} - S_D} H_{ab}(\alpha, \beta_a, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \phi_{\eta_c}^v(x_2, b_2) \phi_B^+(x_1, b_1) \eta_2^+ \left[m_b \eta_3^- + \sqrt{2} \eta(x_1 - x_2) + \sqrt{2} m_3^2(x_1 - x_3) \right] \right. \\
& \quad + \phi_{\eta_c}^v(x_2, b_2) \phi_B^-(x_1, b_1) \eta_2^- \left[m_b \eta_3^+ + \sqrt{2} \eta(x_1 - x_2) + \sqrt{2} m_3^2(x_1 - x_3) \right] \\
& \quad + \phi_{\eta_c}^s(x_2, b_2) \phi_B^+(x_1, b_1) m_2 m_3 \left[m_b + \frac{1}{2} m_1 x_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \quad \left. + \phi_{\eta_c}^s(x_2, b_2) \phi_B^-(x_1, b_1) m_2 m_3 \left[m_b + \frac{1}{2} m_1 x_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{C2}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_b = & \frac{32\pi^2 C_F}{\sqrt{2N}} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\
& \times \alpha_s(t_b) C_1(t_b) e^{-S_B - S_{\eta_c} - S_D} H_{ab}(\alpha, \beta_b, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \sqrt{2} \phi_{\eta_c}^v(x_2, b_2) \left[\eta_3^+ \phi_B^+(x_1, b_1) + \eta_3^- \phi_B^-(x_1, b_1) \right] \left[\eta(x_1 - \bar{x}_3) + m_2^2(x_1 - \bar{x}_2) \right] \right. \\
& \quad - \phi_{\eta_c}^s(x_2, b_2) \phi_B^+(x_1, b_1) m_2 m_3 \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \quad \left. - \phi_{\eta_c}^s(x_2, b_2) \phi_B^-(x_1, b_1) m_2 m_3 \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{C3}
\end{aligned}$$

$$\mathcal{A}_c = \frac{8\pi^2 C_F}{N} m_1 f_{B_s} \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty b_3 \mathbf{d}b_3$$

$$\begin{aligned}
& \times \alpha_s(t_c) [C_1(t_c) + NC_2(t_c)] e^{-S_{\eta_c} - S_D} H_{cd}(\alpha, \beta_c, b_2, b_3) \phi_D(x_3, b_3) \\
& \times \left\{ \left[\sqrt{2} \eta_2^+ x_2 - m_1 \right] \left[\eta_2^+ \eta_3^- \phi_{\eta_c}^v(x_2, b_2) + m_2 m_3 \phi_{\eta_c}^s(x_2, b_2) \right] \right. \\
& \quad \left. + \left[\sqrt{2} \eta_2^- x_2 - m_1 \right] \left[\eta_2^- \eta_3^+ \phi_{\eta_c}^v(x_2, b_2) + m_2 m_3 \phi_{\eta_c}^s(x_2, b_2) \right] \right\}, \tag{C4}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_d &= \frac{8\pi^2 C_F}{N} m_1 f_{B_s} \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty b_3 \mathbf{d}b_3 \\
& \times \alpha_s(t_d) [C_1(t_d) + NC_2(t_d)] e^{-S_{\eta_c} - S_D} H_{cd}(\alpha, \beta_d, b_3, b_2) \phi_D(x_3, b_3) \\
& \times \left\{ \left[m_1 - \sqrt{2} \eta_3^+ x_3 \right] \left[\eta_3^+ \eta_2^- \phi_{\eta_c}^v(x_2, b_2) + m_2 m_3 \phi_{\eta_c}^s(x_2, b_2) \right] \right. \\
& \quad + \left[m_1 - \sqrt{2} \eta_3^- x_3 \right] \left[\eta_3^- \eta_2^+ \phi_{\eta_c}^v(x_2, b_2) + m_2 m_3 \phi_{\eta_c}^s(x_2, b_2) \right] \\
& \quad \left. - m_3 m_c E_2 \phi_{\eta_c}^v(x_2, b_2) - 2 m_2 m_c E_3 \phi_{\eta_c}^s(x_2, b_2) \right\}, \tag{C5}
\end{aligned}$$

Appendix D: The amplitudes for $B_s \rightarrow \eta_c D$ decay

$$\mathcal{A}(B_s \rightarrow \eta_c D) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} \sum_{i=a,b,c,d} \mathcal{A}_i \tag{D1}$$

$$\begin{aligned}
\mathcal{A}_a &= \frac{32\pi^2 C_F}{\sqrt{2}N} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\
& \times \alpha_s(t_a) C_1(t_a) e^{-S_B - S_{\eta_c} - S_D} H_{ab}(\alpha, \beta_a, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \phi_{\eta_c}^v(x_2, b_2) \phi_B^+(x_1, b_1) \eta_3^+ \left[m_b \eta_2^- + \sqrt{2} \eta(x_1 - x_3) + \sqrt{2} m_2^2(x_1 - x_2) \right] \right. \\
& \quad + \phi_{\eta_c}^v(x_2, b_2) \phi_B^-(x_1, b_1) \eta_3^- \left[m_b \eta_2^+ + \sqrt{2} \eta(x_1 - x_3) + \sqrt{2} m_2^2(x_1 - x_2) \right] \\
& \quad + \phi_{\eta_c}^s(x_2, b_2) \phi_B^+(x_1, b_1) m_2 m_3 \left[m_b + \frac{1}{2} m_1 x_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \quad \left. + \phi_{\eta_c}^s(x_2, b_2) \phi_B^-(x_1, b_1) m_2 m_3 \left[m_b + \frac{1}{2} m_1 x_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{D2}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_b &= \frac{32\pi^2 C_F}{\sqrt{2}N} m_1 \int_0^1 \mathbf{d}x_1 \int_0^1 \mathbf{d}x_2 \int_0^1 \mathbf{d}x_3 \int_0^\infty b_1 \mathbf{d}b_1 \int_0^\infty b_2 \mathbf{d}b_2 \int_0^\infty \mathbf{d}b_3 \\
& \times \alpha_s(t_b) C_1(t_b) e^{-S_B - S_{\eta_c} - S_D} H_{ab}(\alpha, \beta_b, b_1, b_2) \phi_D(x_3, b_3) \delta(b_2 - b_3) \\
& \times \left\{ \sqrt{2} \phi_{\eta_c}^v(x_2, b_2) \left[\eta_2^+ \phi_B^+(x_1, b_1) + \eta_2^- \phi_B^-(x_1, b_1) \right] \left[\eta(x_1 - \bar{x}_2) + m_3^2(x_1 - \bar{x}_3) \right] \right. \\
& \quad - \phi_{\eta_c}^s(x_2, b_2) \phi_B^+(x_1, b_1) m_2 m_3 \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^+ x_2 - \frac{1}{\sqrt{2}} \eta_3^+ x_3 \right] \\
& \quad \left. - \phi_{\eta_c}^s(x_2, b_2) \phi_B^-(x_1, b_1) m_2 m_3 \left[\frac{1}{2} m_1 \bar{x}_1 - \frac{1}{\sqrt{2}} \eta_2^- x_2 - \frac{1}{\sqrt{2}} \eta_3^- x_3 \right] \right\}, \tag{D3}
\end{aligned}$$

$$\mathcal{A}_c = -\mathcal{A}_d(B_s \rightarrow \eta_c \overline{D}), \quad (\text{D4})$$

$$\mathcal{A}_d = -\mathcal{A}_c(B_s \rightarrow \eta_c \overline{D}) \quad (\text{D5})$$

Appendix E: Some formula in the decay amplitudes

The S_B ($S_{\eta_c, J/\psi}$ and S_D) in the factor e^{-S_B} ($e^{-S_{\eta_c, J/\psi}}$ and e^{-S_D}) is defined as

$$S_B(t) = s(x_1 p_1^+, b_1) + 2 \int_{1/b_1}^t \frac{d\mu}{d\mu} \gamma_q(\mu), \quad (\text{E1})$$

$$S_{\eta_c, J/\psi}(t) = s(x_2 p_2^+, b_2) + 2 \int_{1/b_2}^t \frac{d\mu}{d\mu} \gamma_q(\mu), \quad (\text{E2})$$

$$S_D(t) = s(x_3 p_3^-, b_3) + 2 \int_{1/b_3}^t \frac{d\mu}{d\mu} \gamma_q(\mu), \quad (\text{E3})$$

where the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The expression of $s(Q, b)$ is given in [6].

$$H_{ab}(\alpha, \beta, b_1, b_2) = K_0(b_1 \sqrt{\beta}) \{ \theta(b_1 - b_2) H_0^{(1)}(b_1 \sqrt{\alpha}) J_0(b_2 \sqrt{\alpha}) + (b_1 \leftrightarrow b_2) \}, \quad (\text{E4})$$

$$H_{cd}(\alpha, \beta, b_i, b_j) = H_0^{(1)}(b_j \sqrt{\alpha}) \{ \theta(b_i - b_j) K_0(b_i \sqrt{\beta}) I_0(b_j \sqrt{\beta}) + (b_i \leftrightarrow b_j) \}, \quad (\text{E5})$$

The virtualities of internal gluons (α) and quarks (β_i), and the typical scale t_i are defined as (where the subscript $i = a, b, c, d$ corresponds to the Fig.1)

$$\alpha = \bar{x}_2^2 m_2^2 + \bar{x}_3^2 m_3^2 + 2\bar{x}_2 \bar{x}_3 \eta, \quad (\text{E6})$$

$$\begin{aligned} -\beta_a &= m_1^2(x_1 - x_2)(x_1 - x_3) - m_b^2 \\ &+ m_2^2(x_2 - x_1)(x_2 - x_3) \\ &+ m_3^2(x_3 - x_1)(x_3 - x_2), \end{aligned} \quad (\text{E7})$$

$$\begin{aligned} -\beta_b &= m_1^2(x_1 - \bar{x}_2)(x_1 - \bar{x}_3) \\ &+ m_2^2(\bar{x}_2 - x_1)(\bar{x}_2 - \bar{x}_3) \\ &+ m_3^2(\bar{x}_3 - x_1)(\bar{x}_3 - \bar{x}_2), \end{aligned} \quad (\text{E8})$$

$$-\beta_c = m_1^2 + x_2^2 m_2^2 - x_2(m_1^2 + m_2^2 - m_3^2), \quad (\text{E9})$$

$$\begin{aligned} -\beta_d &= m_1^2 + x_3^2 m_3^2 - m_c^2 \\ &- x_3(m_1^2 - m_2^2 + m_3^2), \end{aligned} \quad (\text{E10})$$

$$t_{a(b)} = \max(\sqrt{\alpha}, \sqrt{|\beta_{a(b)}|}, 1/b_1, 1/b_2, 1/b_3), \quad (\text{E11})$$

$$t_{c(d)} = \max(\sqrt{\alpha}, \sqrt{|\beta_{c(d)}|}, 1/b_2, 1/b_3). \quad (\text{E12})$$

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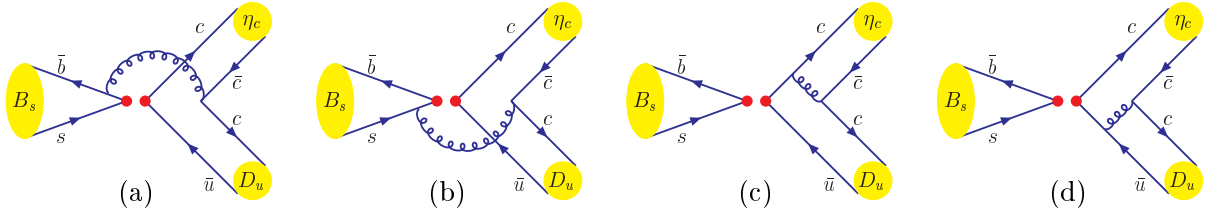
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TABLE I: input parameters for $B_s \rightarrow \eta_c(J/\psi)D$ decay

parameter	value	reference
mass of B_s meson	$m_{B_s} = 5366.77 \pm 0.24$ MeV	[9]
mass of J/ψ meson	$m_{J/\psi} = 3096.916 \pm 0.011$ MeV	[9]
mass of η_c meson	$m_{\eta_c} = 2981.0 \pm 1.1$ MeV	[9]
mass of D meson	$m_D = 1864.86 \pm 0.13$ MeV	[9]
mass of b quark	$m_b = 4.18 \pm 0.03$ GeV	[9]
mass of c quark	$m_c = 1.275 \pm 0.025$ GeV	[9]
lifetime of B_s meson	$\tau_{B_s} = 1.497 \pm 0.015$ ps	[9]
decay constant of D meson	$f_D = 206.7 \pm 8.9$ MeV	[9]
decay constant of J/ψ meson	$f_{J/\psi} = 405 \pm 6$ MeV	[31]
decay constant of η_c meson	$f_{\eta_c} = 394.7 \pm 2.4$ MeV	[32]
decay constant of B_s meson	$f_{B_s} = 227.6 \pm 5.0$ MeV	[33]
Wolfenstein parameters	$A = 0.811^{+0.022}_{-0.012}$	[9]
	$\lambda = 0.22535 \pm 0.00065$	[9]
	$\bar{\rho} = 0.131^{+0.026}_{-0.013}$	[9]
	$\bar{\eta} = 0.345^{+0.013}_{-0.014}$	[9]

TABLE II: Branching ratio for $B_s \rightarrow \eta_c(J/\psi)D$ decay with different models of WFs.

		GN	KLS	KKQT
$\mathcal{BR}(B_s \rightarrow J/\psi \bar{D}) \times 10^8$	O	$1.54^{+0.94+0.61+0.13+0.05}_{-0.61-0.34-0.12-0.05}$	$3.02^{+0.84+0.96+0.11+0.02}_{-0.91-0.62-0.08-0.03}$	$2.92^{+1.63+0.96+0.08+0.05}_{-1.91-0.61-0.07-0.07}$
	C	$2.83^{+1.02+3.56+0.09+0.25}_{-0.74-1.73-0.06-0.33}$	$4.37^{+0.72+4.76+0.19+0.31}_{-0.89-2.41-0.16-0.42}$	$4.00^{+1.58+4.88+0.16+0.30}_{-2.52-2.39-0.14-0.42}$
$\mathcal{BR}(B_s \rightarrow J/\psi D) \times 10^9$	O	$4.34^{+1.37+0.27+0.08+0.31}_{-1.40-0.16-0.07-0.33}$	$6.28^{+0.63+0.32+0.17+0.38}_{-0.68-0.22-0.14-0.41}$	$7.09^{+0.32+1.00+0.12+0.42}_{-1.84-0.66-0.09-0.46}$
	C	$7.08^{+0.65+1.45+0.07+0.36}_{-1.55-0.98-0.05-0.33}$	$9.08^{+0.61+2.08+0.27+0.35}_{-1.54-1.30-0.21-0.30}$	$15.07^{+0.78+5.72+0.10+0.50}_{-7.15-3.48-0.03-0.45}$
$\mathcal{BR}(B_s \rightarrow \eta_c \bar{D}) \times 10^7$	O	$4.09^{+0.87+0.67+0.31+0.07}_{-0.84-0.49-0.27-0.09}$	$5.30^{+0.55+0.89+0.36+0.09}_{-0.68-0.64-0.31-0.11}$	$5.36^{+0.87+0.89+0.37+0.07}_{-1.58-0.64-0.32-0.09}$
	C	$5.36^{+1.14+3.57+0.17+0.19}_{-1.05-1.98-0.11-0.24}$	$7.22^{+0.63+5.02+0.16+0.24}_{-0.98-2.74-0.07-0.31}$	$7.24^{+1.14+4.77+0.19+0.21}_{-2.74-2.65-0.10-0.27}$
$\mathcal{BR}(B_s \rightarrow \eta_c D) \times 10^8$	O	$9.48^{+1.08+0.21+0.12+0.08}_{-1.43-0.26-0.10-0.00}$	$11.43^{+0.06+0.21+0.27+0.07}_{-0.42-0.26-0.24-0.00}$	$12.46^{+0.02+0.58+0.22+0.09}_{-1.95-0.55-0.19-0.00}$
	C	$7.96^{+0.29+0.74+0.24+0.09}_{-0.86-0.66-0.22-0.01}$	$9.39^{+0.34+1.30+0.42+0.05}_{-1.02-0.90-0.38-0.00}$	$11.93^{+0.15+2.31+0.33+0.10}_{-3.43-1.65-0.28-0.00}$


 FIG. 1: Feynman diagrams for $B_s \rightarrow \eta_c D$ decay within the pQCD framework